

Wednesday 28 June 2017 – Morning

A2 GCE MATHEMATICS

4731/01 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

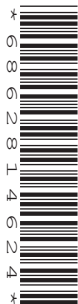
OCR supplied materials:

- Printed Answer Book 4731/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

- 1 A uniform rod with centre C has mass $2M$ and length $4a$. The rod is free to rotate in a vertical plane about a smooth fixed horizontal axis passing through a point A on the rod, where $AC = ka$ and $0 < k < 2$. The rod is making small oscillations about the equilibrium position with period T .

(i) Show that $T = 2\pi\sqrt{\frac{a}{3g}\left(\frac{4+3k^2}{k}\right)}$. (You may assume the standard formula $T = 2\pi\sqrt{\frac{I}{mgh}}$ for the period of small oscillations of a compound pendulum.) [4]

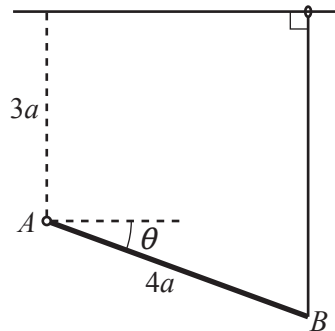
(ii) Hence find the value of k^2 for which the period of oscillations is least. [3]

- 2 A ship S is travelling with constant speed 5 m s^{-1} on a course with bearing 325° . A second ship T observes S when S is 9500 m from T on a bearing of 060° from T . Ship T sets off in pursuit, travelling with constant speed 8.5 m s^{-1} in a straight line.

(i) Find the bearing of the course which T should take in order to intercept S . [4]

(ii) Find the distance travelled by S from the moment that T sets off in pursuit until the point of interception. [5]

3



A uniform rod AB has mass m and length $4a$. The rod can rotate in a vertical plane about a smooth fixed horizontal axis passing through A . One end of a light elastic string of natural length a and modulus of elasticity λmg is attached to B . The other end of the string is attached to a small light ring which slides on a fixed smooth horizontal rail which is in the same vertical plane as the rod. The rail is a vertical distance $3a$ above A . The string is always vertical and the rod makes an angle θ radians with the horizontal, where $0 \leq \theta \leq \frac{1}{2}\pi$ (see diagram).

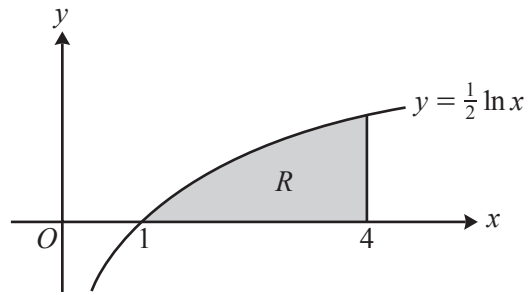
- (i) Taking A as the reference level for gravitational potential energy, find an expression for the total potential energy V of the system, and show that

$$\frac{dV}{d\theta} = 2mga \cos \theta (4\lambda(1 + 2 \sin \theta) - 1). \quad [6]$$

Determine the positions of equilibrium and the nature of their stability in the cases

(ii) $\lambda > \frac{1}{12}$, [9]

(iii) $\lambda < \frac{1}{12}$. [2]



The diagram shows the curve with equation $y = \frac{1}{2} \ln x$. The region R , shaded in the diagram, is bounded by the curve, the x -axis and the line $x = 4$. A uniform solid of revolution is formed by rotating R completely about the y -axis to form a solid of volume V .

(i) Show that $V = \frac{1}{4}\pi(64 \ln 2 - 15)$. [4]

(ii) Find the exact y -coordinate of the centre of mass of the solid. [7]

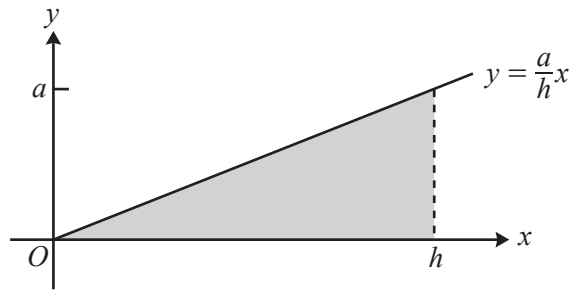


Fig. 1

Fig. 1 shows part of the line $y = \frac{a}{h}x$, where a and h are constants. The shaded region bounded by the line, the x -axis and the line $x = h$ is rotated about the x -axis to form a uniform solid cone of base radius a , height h and volume $\frac{1}{3}\pi a^2 h$. The mass of the cone is M .

- (i) Show by integration that the moment of inertia of the cone about the y -axis is $\frac{3}{20}M(a^2 + 4h^2)$. (You may assume the standard formula $\frac{1}{4}mr^2$ for the moment of inertia of a uniform disc about a diameter.)

[7]

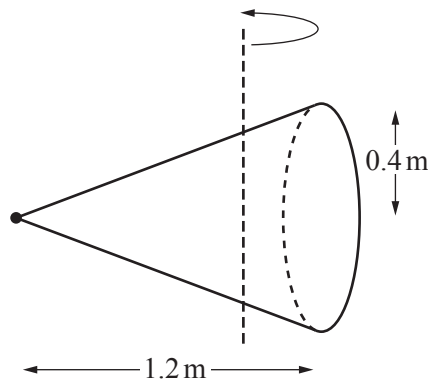
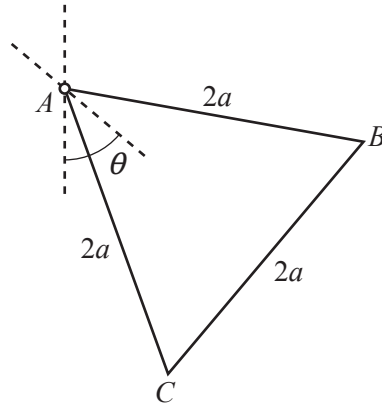


Fig. 2

A uniform solid cone has mass 3 kg, base radius 0.4 m and height 1.2 m. The cone can rotate about a fixed vertical axis passing through its centre of mass with the axis of the cone moving in a horizontal plane. The cone is rotating about this vertical axis at an angular speed of 9.6 rad s^{-1} . A stationary particle of mass m kg becomes attached to the vertex of the cone (see Fig. 2). The particle being attached to the cone causes the angular speed to change instantaneously from 9.6 rad s^{-1} to 7.8 rad s^{-1} .

- (ii) Find the value of m .

[5]



A triangular frame ABC consists of three uniform rods AB , BC and CA , rigidly joined at A , B and C . Each rod has mass m and length $2a$. The frame is free to rotate in a vertical plane about a fixed horizontal axis passing through A . The frame is initially held such that the axis of symmetry through A is vertical and BC is below the level of A . The frame starts to rotate with an initial angular speed of ω and at time t the angle between the axis of symmetry through A and the vertical is θ (see diagram).

(i) Show that the moment of inertia of the frame about the axis through A is $6ma^2$. [3]

(ii) Show that the angular speed $\dot{\theta}$ of the frame when it has turned through an angle θ satisfies

$$a\dot{\theta}^2 = a\omega^2 - kg\sqrt{3}(1 - \cos\theta),$$

stating the exact value of the constant k .

Hence find, in terms of a and g , the set of values of ω^2 for which the frame makes complete revolutions. [5]

At an instant when $\theta = \frac{1}{6}\pi$, the force acting on the frame at A has magnitude F .

(iii) Given that $\omega^2 = \frac{2g}{a\sqrt{3}}$, find F in terms of m and g . [8]

END OF QUESTION PAPER

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